Calculating the Rank of a Matrix for Determimants

http://www.vitutor.com/alg/determinants/matrix_rank.html

$$B = \begin{pmatrix} 2 & 1 & 3 & 2 \\ 3 & 2 & 5 & 1 \\ -1 & 1 & 0 & -7 \\ 3 & -2 & 1 & 17 \\ 0 & 1 & 1 & -4 \end{pmatrix}$$

1. A line can eliminated a if:

All the coefficients are zeros.

There are two equal lines.

A line is proportional to another.

A line is a linear combination of others.

The third column can be deleted because it is a linear combination of the first two: $\mathbf{c_3 = c_1 + c_2}$

$$\begin{pmatrix} 2 & 1 & 3 & 2 \\ 3 & 2 & 5 & 1 \\ -1 & 1 & 0 & -7 \\ 3 & -2 & 1 & 17 \\ 0 & 1 & 1 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & -7 \\ 3 & -2 & 17 \\ 0 & 1 & -4 \end{pmatrix}$$

2. Check to see if the rank is 1, for it must be satisfied that the element of the matrix is not zero and therefore its determinant is not zero.

$\mathbf{|2| = 2 \neq 0}$

3. The matrix will have a rank of 2 if there is a square submatrix of order 2 and its determinant is not zero.

$$\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1 \neq 0$$

4. The matrix will have a rank of 3 if there is a square submatrix of order 3 and its determinant is not zero.

$$\begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & -7 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 3 & -2 & 17 \end{vmatrix} = 0 \qquad \left| \begin{matrix} 2 \\ 3 \\ 0 \end{matrix} \right.$$

As all the determinants of the submatrices are zero, it does not have a rank of 3, therefore **r(B) = 2**.

If the matrix had a rank of 3 and there was a submatrix of order 4, whose determinant was not zero, it would have had a rank of 4. In the same way as shown above, check to see if there is a range greater than 4.

Examples

1. Calculate the rank of the matrix:

$$A = \begin{pmatrix} 2 & 3 & 1 & 6 \\ -1 & -2 & 0 & -3 \\ 3 & 5 & 1 & 9 \end{pmatrix}$$

$|2| = 2 \neq 0$

$$\begin{vmatrix} 2 & 3 \\ -1 & -2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 0 \\ 3 & 5 & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 3 & 6 \\ -1 & -2 & -3 \\ 3 & 5 & 9 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 6 & 1 \\ -1 & -3 & 0 \\ 3 & 9 & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} 6 & 3 & 1 \\ -3 & -2 & 0 \\ 9 & 5 & 1 \end{vmatrix} = 0$$

r(A) = 2

2. Calculate the rank of the matrix:

$$B = \begin{pmatrix} 3 & 2 & 4 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & -1 & 1 & 3 \\ -1 & 2 & 4 & 2 \\ 0 & 1 & -1 & 3 \end{pmatrix}$$

$$|3| = 3 \neq 0 \qquad \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1 \neq 0 \qquad \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = -$$

$$= \begin{vmatrix} 3 & 2 & 4 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & -1 & 1 & 3 \\ -1 & 2 & 4 & 2 \\ 0 & 1 & -1 & 3 \end{vmatrix} = -99 \neq 0$$

r(B) = 4

3. Calculate the rank of the matrix:

$$C = \begin{pmatrix} 1 & 1 & 0 & 3 & -1 \\ 1 & 2 & 0 & 3 & 0 \\ 1 & 3 & 0 & 3 & 1 \\ 1 & 4 & 0 & 3 & 2 \\ 1 & 5 & 0 & 3 & 3 \\ 1 & 6 & 0 & 3 & 4 \end{pmatrix}$$

Remove the third column as it is zero, the fourth because it is proportional to the first and the fifth because it is the linear combination of the first and second: $c_5 = -2 \cdot c_1 + c_2$

$$\begin{pmatrix} 1 & 1 & 0 & 3 & -1 \\ 1 & 2 & 0 & 3 & 0 \\ 1 & 3 & 0 & 3 & 1 \\ 1 & 4 & 0 & 3 & 2 \\ 1 & 5 & 0 & 3 & 3 \\ 1 & 6 & 0 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{pmatrix}$$

$$|1| = 1 \neq 0 \qquad \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \neq 0$$

r(C) = 2