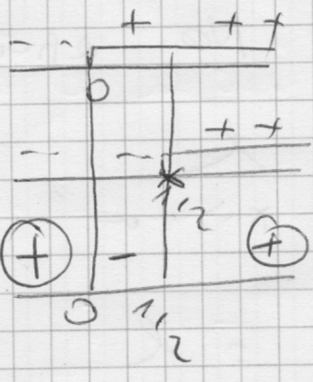
1) $f(x) = \operatorname{arctg}\sqrt{\frac{x}{2x-1}} + \arcsin(3^x-2)$

$$\begin{cases} \frac{x}{2x-1} \geq 0 \\ -1 \leq 3^x - 2 \leq 1 \\ 2x-1 \neq 0 \end{cases}$$

1) $\frac{x}{2x-1} \geq 0$ $\qquad N \geq 0 \qquad x \geq 0$

$\qquad\qquad\qquad D > 0 \qquad x > 1/2$

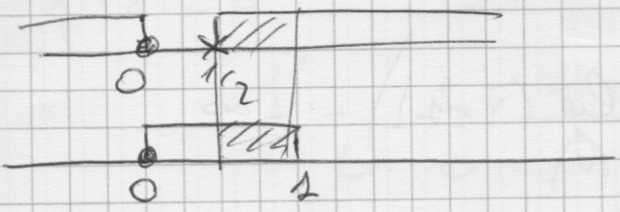


$\Rightarrow \quad x \leq 0, \quad x > 1/2$

2) $3^x - 2 \leq 1 \iff 3^x \leq 3 \iff x \leq 1$

3) $3^x - 2 \geq -1 \iff 3^x \geq 1 = 3^0 \iff x \geq 0$

Tornando al sistema $\begin{cases} x \leq 0, \ x > 1/2 \\ x \leq 1, \ x \geq 0 \\ x \neq 1/2 \end{cases}$



$\frac{1}{2} < x \leq 1 \ ; \ x = 0$

2) $f(x) = \frac{x^2}{2} + \lg|x-1|$

CE $|x-1| > 0 \iff x \neq 1$

$f(0) = 0$ ~~Resta~~

f né pari né dispari

Asintoti

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\left(\frac{x^2}{2} + \lg(x-1)\right) = +\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(\frac{x^2}{2} + \lg(1-x)\right) = +\infty$$

No asintoti orizzontali

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\frac{x^2}{2} + \lg(x-1)}{x} =$$

$$= \lim_{x\to+\infty}\left(\frac{x}{2} + \underbrace{\frac{\lg(x-1)}{x}}_{\to 0}\right) = +\infty$$

$$\lim_{x\to-\infty} \frac{\left(\frac{x^2}{2} + \lg(1-x)\right)}{x} = \lim_{x\to-\infty}\left(\frac{x}{2} + \underbrace{\frac{\lg(1-x)}{x}}_{\to 0}\right)$$

$= -\infty$ No asintoti obliqui

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+}\left(\frac{x^2}{2} + \lg(x-1)\right) = -\infty$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-}\left(\frac{x^2}{2} + \lg(1-x)\right) = -\infty$$

$x=1$ Asintoto verticale

Studio di crescenza e decrescenza

$$f'(x) = \frac{2x}{2} + \frac{1}{|x-1|}\,\frac{|x-1|}{x-1} = x + \frac{1}{x-1} =$$

$$= \frac{x^2-x+1}{x-1} .$$ $f'(x)$ esiste nel C.E. di f

N. $x^2-x+1 \geq 0$

$\Delta = 1-4 = -3 < 0 \Rightarrow x^2-x+1>0 \quad \forall x \in \mathbb{R}$

D. $x-1>0$ per $x>1$

$\Rightarrow$

		1	
f'	$-$		$+$
f	$\searrow$		$\nearrow$

Studio di convessità e concavità

$$f''(x) = \frac{(2x-1)(x-1) - (x^2-x+1)}{(x-1)^2} = \frac{2x^2-2x-\cancel{x}+\cancel{1}-x^2+\cancel{x}-\cancel{1}}{(x-1)^2}$$

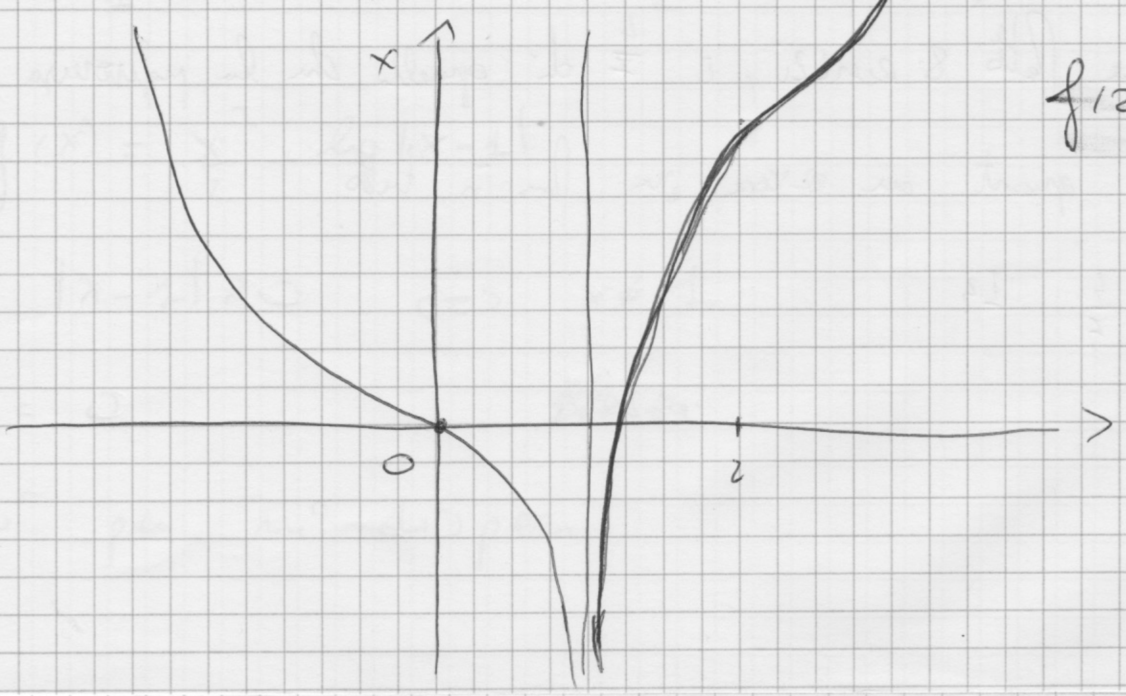
$$= \frac{x^2-2x}{(x-1)^2}$$

$f''(x) \geq 0 \iff x^2-2x \geq 0 \iff x \leq 0 ;\ x \geq 2$

		0		2	
f''	$+$		$-$		$+$
	$\cup$		$\cap$		$\cup$

$x=0,\ x=2$ punti di flesso

$f(2)=2$



3) $\lim_{x \to 0} \dfrac{2-2\cos x - x^2}{x^4}$

$f(x) = 2 - 2\cos x - x^2$ $\qquad f(0)=0$

$f'(x) = 2\sin x - 2x$ $\qquad f'(0)=0$

$f''(x) = 2\cos x - 2$ $\qquad f''(0)=0$

$f'''(x) = -2\sin x$ $\qquad f'''(0)=0$

$f^{IV}(x) = -2\cos x$ $\qquad f^{IV}(0) = -2$

$$\lim_{x \to 0} \frac{2-2\cos x - x^2}{x^4} = \lim_{x \to 0} \frac{\frac{-2}{4!}x^4 + o(x^4)}{x^4}$$

$$= -\frac{2}{4 \cdot 3 \cdot 2} = -\frac{1}{12}$$

4) $f(x) = \begin{cases} \dfrac{e^{\sin x} - \cos x}{\lg(x+1)} & x > 0 \\ \operatorname{arctg}(x+1) & x = 0 \end{cases}$

$$\lim_{x \to 0^+} \frac{e^{\sin x} - \cos x}{\lg(x+1)} = \frac{0}{0} = \lim_{x \to 0^+} \frac{e^{\sin x}\cos x + \sin x}{\frac{1}{x+1}} =$$

$= \cancel{\ }$ 1

$$\lim_{x \to 0^-} \operatorname{arctg}(x+1) = \pi/4 = f(0)$$

f continua in $\mathbb{R} \setminus \{0\}$

$x = 0$ pto di discontinuità di I specie