Compito di Analisi Matematica I

11/06/2018

1) $f(x) = \sqrt{\lg x} + \arcsin \dfrac{x}{x+1}$

CE $\begin{cases} \lg x \geq 0 \\ x > 0 \\ -1 \leq \dfrac{x}{x+1} \leq 1 \end{cases} \iff \begin{cases} x \geq 1 \\ x > 0 \\ -1 \leq \dfrac{x}{x+1} \leq 1 \end{cases}$

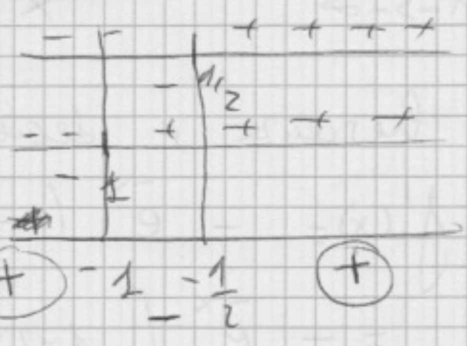
I) $\dfrac{x}{x+1} \leq 1 \iff \dfrac{x}{x+1} - 1 \leq 0 \iff \dfrac{x - x - 1}{x+1} \leq 0$

$\iff \dfrac{-1}{x+1} \leq 0 \iff \dfrac{1}{x+1} \geq 0 \iff \boxed{x > -1}$

II) $\dfrac{x}{x+1} \geq -1 \iff \dfrac{x}{x+1} + 1 \geq 0 \iff \dfrac{x+x+1}{x+1} \geq 0$

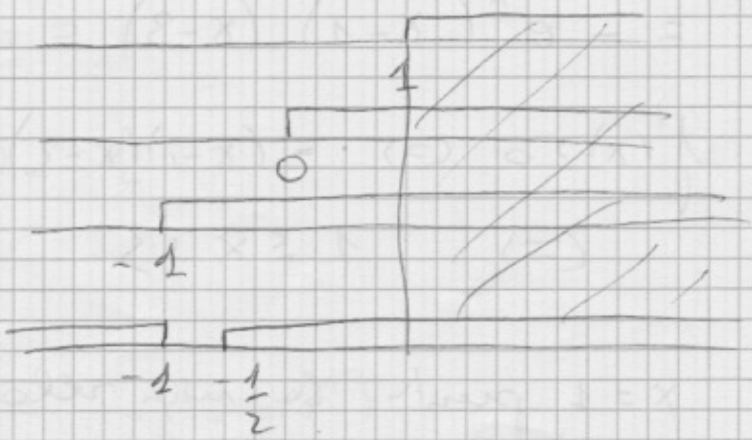
$\iff \dfrac{2x+1}{x+1} \geq 0$ N $2x+1 \geq 0$

D $x+1 > 0$



$\boxed{x < -1 ;\ x \geq -\frac{1}{2}}$

$\begin{cases} x \geq 1 \\ x > 0 \\ x > -1 \\ x < -1 ;\ x \geq -\frac{1}{2} \end{cases}$



CE $\boxed{x \geq 1}$

2) $f(x) = e^{-x}(1-x)^2$

C.E. $= \mathbb{R}$, f non è pari né dispari

$f(x) \geq 0 \quad \forall x \in \mathbb{R}$

$x = 0 \iff y = 1$ $\qquad (0,1)$

$y = 0 \iff x = 1$ $\qquad (1,0)$

$$\lim_{x \to +\infty} e^{-x}(1-x)^2 = (0 \cdot \infty) = \lim_{x \to +\infty} \frac{(1-x)^2}{e^x} = 0$$

per il confronto tra infiniti

$y = 0$ asintoto orizzontale destro

$$\lim_{x \to -\infty} e^{-x}(1-x)^2 = +\infty$$

Non è asintoto orizzontale

$$\lim_{x \to -\infty} e^{-x}\frac{(1-x)^2}{x} = -\infty$$

né asintoto obliquo a sinistra

Crescenza e decrescenza

$$f'(x) = -e^{-x}(1-x)^2 + e^{-x}\,2(1-x)(-1) =$$

$$= -e^{-x}\left[1-2x+x^2+2-2x\right] = -e^{-x}\left[x^2-4x+3\right]$$

$$= -e^{-x}(x-1)(x-3)$$

$\frac{\Delta}{4} = 4-3 = 1$

$x = 2 \mp 1 = \begin{array}{l} 1 \\ 3 \end{array}$

$f'(x) \geq 0 \iff -(x-1)(x-3) \geq 0$

$\iff 1 \leq x \leq 3$

f	$\searrow$		$\nearrow$		$\searrow$
f'	$-$	1	$+$	3	$-$

$x = 1$ punto di min relativo $\qquad$ C.E.$(f') = \mathbb{R}$

$x = 3$ punto di Max relativo

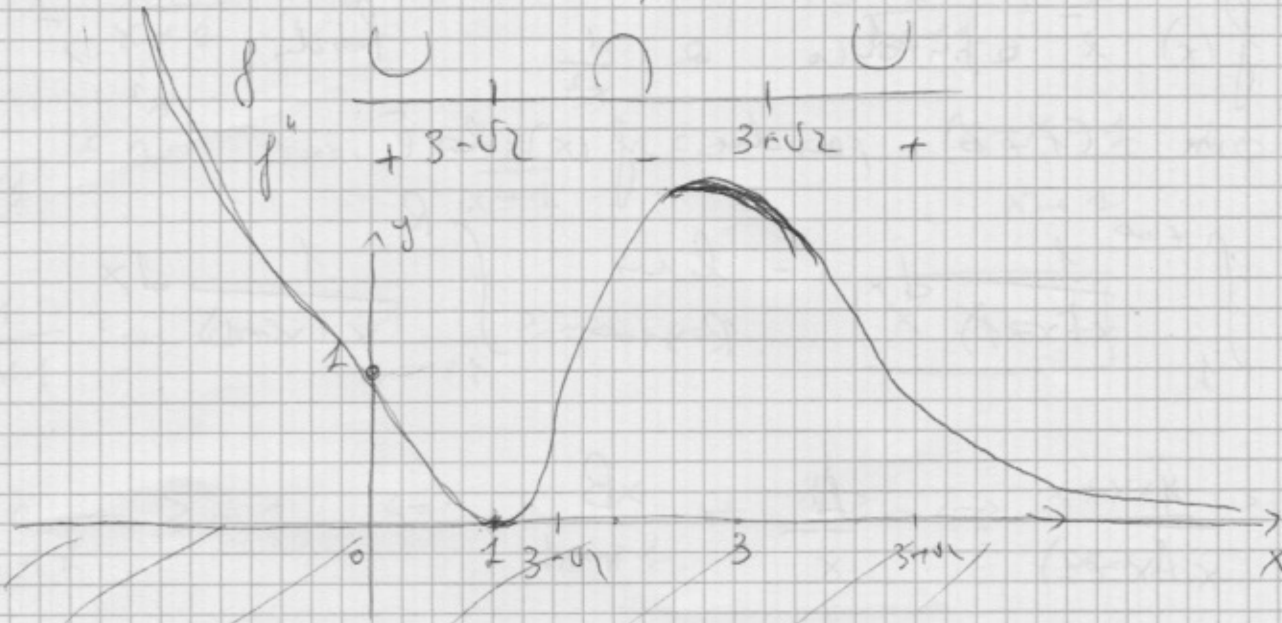
Concavità e convessità

$$f''(x) = e^{-x}\left[x^2-4x+3\right] - e^{-x}\left[2x-4\right] =$$

$$= e^{-x}\left[x^2-4x+3-2x+4\right] = e^{-x}\left[x^2-6x+7\right]$$

$$\frac{\Delta}{4} = 9-7=2 \qquad x = 3 \mp \sqrt{2}$$

$$f''(x) \geq 0 \iff x \leq 3-\sqrt{2}\ ;\ x \geq 3+\sqrt{2}$$



3) $$\lim_{x\to 0} \frac{\lg(\cos x)}{x^2}$$

$f(x) = \lg(\cos x)$ $\qquad f(0) = 0$

$f'(x) = -\frac{\sin x}{\cos x} = -\operatorname{tg} x$ $\qquad f'(0) = 0$

$f''(x) = -(1+\operatorname{tg}^2 x)$ $\qquad f''(0) = -1$

$$f(x) = -\frac{x^2}{2} + o(x^2)$$

$$\lim_{x\to 0} \frac{\lg(\cos x)}{x^2} = \lim_{x\to 0} \frac{-\frac{x^2}{2}+o(x^2)}{x^2} = -\frac{1}{2}$$

46) $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx = \operatorname{arctg} e^x + c$

45) $f(x) = \dfrac{1}{x(x+1)}$ $\qquad \displaystyle\lim_{x\to+\infty} \frac{\frac{1}{x^2+x}}{\frac{1}{x^2}} = 1$

$f(x)$ è asintotica a $\frac{1}{x^2}$. Poiché $\frac{1}{x^2}$ è integrabile in $(1,+\infty)$, anche $f(x)$ lo è.

$$\int_1^{+\infty} \frac{1}{x(x+1)}\,dx = \lim_{R\to+\infty} \int_1^R \frac{1}{x(x+1)}\,dx$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\frac{1}{x(x+1)} = \frac{Ax+A+Bx}{x(x+1)}$$

$$\begin{cases} A = 1 \\ A+B = 0 \end{cases} \qquad \begin{cases} A = 1 \\ B = -1 \end{cases}$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

$$\int_1^{+\infty} \frac{1}{x(x+1)}\,dx = \lim_{R\to+\infty} \left[\int_1^R \frac{1}{x}\,dx - \int_1^R \frac{1}{x+1}\,dx\right]$$

$$= \lim_{R\to+\infty} \left\{ \left[\lg x\right]_1^R - \left[\lg|x+1|\right]_1^R \right\} =$$

$$= \lim_{R \to +\infty} \left\{ \lg R - \lg (R+1) \right\} + \lg 2$$

$$= \lim_{R \to +\infty} \left[\lg 2 + \lg \frac{R}{R+1} \right] = \lg 2$$

5) $$\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$$

Studiamo l'assoluta convergenza

$$\sum_{n=1}^{\infty} \left| \frac{\sin n}{2^n} \right| = \sum_{n=1}^{\infty} \frac{|\sin n|}{2^n}$$

$\frac{|\sin n|}{2^n} \leq \frac{1}{2^n}$ => per il criterio del confronto,

poiché $\sum_{n=1}^{\infty} \frac{1}{2^n}$ è convergente => anche

$\sum_{n=1}^{\infty} \frac{|\sin n|}{2^n}$ è convergente $\Leftrightarrow$ $\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$ è

assolutamente convergente $\Rightarrow$ $\boxed{\sum_{n=1}^{\infty} \frac{\sin n}{2^n} \text{ è convergente}}$

6) $z^3 = 3$ Calcoliamo $\sqrt[3]{3}$

$3 = 3 \, (\cos 0 + i \sin 0)$

$$w_k = \sqrt[3]{3} \left[\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3} \right] \quad k = 0, 1, 2$$

$$w_0 = \sqrt[3]{3}$$

$$w_1 = \sqrt[3]{3}\left[\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right] =$$

$$= \sqrt[3]{3}\left[-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right]$$

$$w_2 = \sqrt[3]{3}\left[\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right] =$$

$$= \sqrt[3]{3}\left[-\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right]$$