

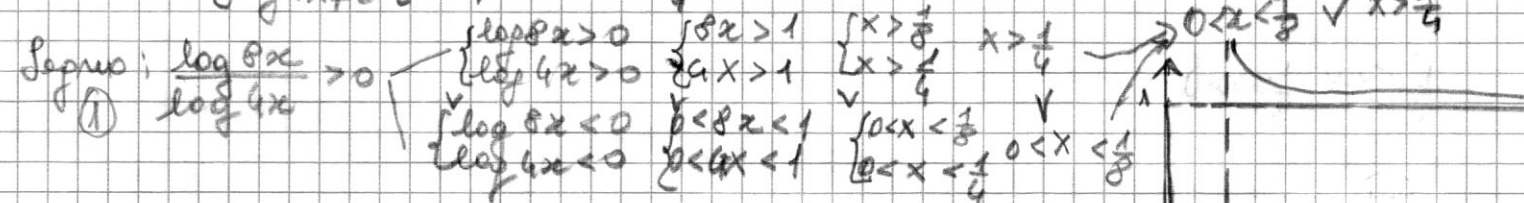


1) $\begin{cases} 4x+x^2 > 0 \\ \log(x^2-5) > 0 \end{cases}$ (a) $\begin{cases} x < -4 \vee x > \sqrt{5} \\ x > \sqrt{5} \vee x < -\sqrt{5} \end{cases}$



$D = [-7, -4] \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, 7]$

2) Dominio $\begin{cases} x > 0 \\ \log 4x \neq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ 4x \neq 1 \\ x \neq 1/4 \end{cases} \Rightarrow D = (0, 1/4) \cup (1/4, +\infty)$



Int. ass: Asse y $\begin{cases} x=0 \\ y = \frac{\log 8x}{\log 4x} \end{cases} \nexists$ perché $0 \notin D$

Asse x $\begin{cases} y=0 \\ y = \frac{\log 8x}{\log 4x} \end{cases} \Rightarrow \begin{cases} \log 8x = 0 \\ 8x = 1 \\ x = 1/8 \end{cases} \Rightarrow A(1/8, 0)$

Comp. estremi: $\lim_{x \rightarrow 0^+} \frac{\log 8x}{\log 4x} \text{ F.I. } \frac{\infty}{\infty}$, Dopo aver verif. Hp, teor. dell'H. poiché $\lim_{x \rightarrow 0^+} \frac{1}{8x} = \infty \Rightarrow \lim_{x \rightarrow 0^+} \frac{\log 8x}{\log 4x} = 1$

$\lim_{x \rightarrow 1/4^-} \frac{\log 8x}{\log 4x} \rightarrow \log 2$
 $\lim_{x \rightarrow 1/4^-} \log 4x \rightarrow 0^-$ perché $\log 4x > 0 \Leftrightarrow 4x > 1 \Rightarrow x > 1/4$; $x = 1/4$ A.V. bilaterale

$\lim_{x \rightarrow 1/4^+} \frac{\log 8x}{\log 4x} \rightarrow \log 2$
 $\lim_{x \rightarrow +\infty} \frac{\log 8x}{\log 4x} \text{ F.I. } \frac{\infty}{\infty}$; si procede con T.H. come per $x \rightarrow 0^+$ $\Rightarrow \lim_{x \rightarrow +\infty} \frac{\log 8x}{\log 4x} = 1 \Rightarrow y = 1$ A.O.

$f'(x) = \frac{1}{x} \cdot 8 \cdot \log 4x - \frac{1}{4x} \cdot 4 \cdot \log 8x = \frac{1}{x} \frac{\log 4x - \log 8x}{(\log 4x)^2} = \frac{1}{x} \frac{\log 1/2}{(\log 4x)^2} = -\frac{1}{x} \frac{\log 2}{(\log 4x)^2}$

$f'(x) < 0 \forall x \in D$ perché $\frac{\log 2}{(\log 4x)^2} > 0 \forall x \in D$; M.B $f''(x) \exists \forall x \in D$.

$f''(x) = \frac{\log 2 \cdot [(\log 4x)^2 + x \cdot 8 \log 4x]}{x^2 (\log 4x)^4} = \frac{\log 2 \cdot \log 4x (\log 4x + 2)}{x^2 (\log 4x)^3}$

$f''(x) > 0 \Leftrightarrow \begin{cases} \log 4x > 0 \\ \log 4x + 2 > 0 \end{cases} \Leftrightarrow \begin{cases} 4x > 1 \\ 4x > e^{-2} \end{cases} \Leftrightarrow \begin{cases} x > 1/4 \\ x > e^{-2}/4 \end{cases} \Rightarrow x > 1/4$
 $f''(x) < 0 \Leftrightarrow \begin{cases} \log 4x < 0 \\ \log 4x + 2 < 0 \end{cases} \Leftrightarrow \begin{cases} 0 < 4x < 1 \\ 4x < e^{-2} \end{cases} \Leftrightarrow \begin{cases} 0 < x < 1/4 \\ 0 < x < e^{-2}/4 \end{cases} \Rightarrow 0 < x < e^{-2}/4$